Investigating the Role Of Working Memory Components In Mathematical Cognition in Children: A Scoping Review

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Mathematics has been long known to be a complicated subject, requiring an advanced set of skills. Each skill learned in mathematics requires different level of effort and goes through a different path of processing when working memory is utilized. Being a leading indicator of children's academic achievement, mathematical cognition is of vital importance and is monitored from a young age, where low performance could be a sign of a learning disability that needs to be addressed. A scoping review of literature was conducted to investigate the fortifying evidence on the importance of working memory in children's mathematical performance, and present the common measures for which future research in this field would need to account. One of the main findings of this research is the strong contribution of working memory to problem solving, whether it is a single- or multi-digit arithmetic in consideration. Verbal working memory will have greater contribution for orally presented problems in contrast to written ones that would interact with visual-spatial sketchpad. A number of measures were agreed on to be good contributors for working memory performance, however some limitations apply on the modeling of working memory which was found to be alleviating some implications; at the same time, providing rich soil for cultivating new research.

INTRODUCTION

The human information processing is a marvelous system that has been studied throughout the ages in an attempt to better understand how we learn, think and interact with the world around us. As the need to study human psychology and performance grew further in the 20th century, different branches of psychology became prominent and actively researched. Cognitive psychology is one of these branches that has a prominent role in the development of education and learning systems. Mathematical cognition is proposed to be one of the leading indicators of how well students perform, whether it is a child or an adult with more focus paid to children being in earlier learning phases. With such importance, difficulties in areas of arithmetic calculation and problem solving might constitute presence of problems regarding children's educational development and later on their daily life experiences.

In order to investigate how working memory is utilized as part of mathematical cognition, basic understanding of working memory needs to be in place. Baddeley (1992) defines working memory as follows:

"a brain system that provides temporary storage and manipulation of the information necessary for such complex cognitive tasks as language comprehension, learning, and reasoning." (p. 556).

The most commonly used model of working memory is the one proposed by Baddeley (2001) (Figure 1). In this model, working memory is composed of the central executive, supplemented by the phonological loop, episodic buffer and visual-spatial sketchpad. The central executive plays the coordinator role and distinguishes resources required for the cognitive task and then assigns subtasks to the three subsystems. The visual-spatial sketchpad is responsible for the storage and articulation of visual-spatial information, while the phonological loop is responsible for verbal information. The episodic buffer assists in understanding working memory interaction with other types of memory, namely the long-term and the short-term memories.

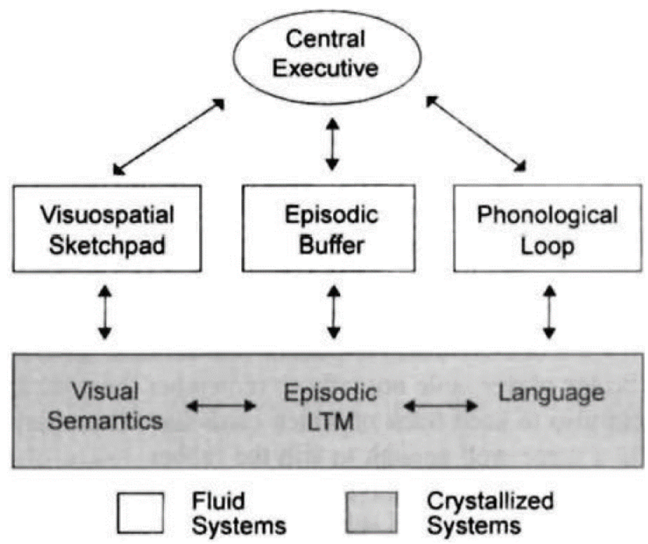

Figure 1. Updated and current working memory model by Baddeley published in 2001.

An important consideration of the aforementioned explanation of working memory model, is that memory here is not just a warehouse for storage of information, but it also manipulates information to perform cognitive tasks; hence the prefix "working". It is the concurrent processing element that distinguishes this type of memory from short and long-term memories, where the latter two have more to do with storage of information.

Throughout this review, arithmetic calculation is assumed to be the main predictor of mathematical cognition performance. The reasoning behind this assumption is that the term mathematical cognition is interchangeably used with numerical cognition, which mainly uses arithmetic calculation as a foundation. Other disciplines of mathematics such as calculus and geometry require a different set of cognitive resources that are marked as advanced for children. These tasks require the use of long-term memory in parallel with working memory making it out of scope for this particular review.

REVIEW OBJECTIVE

In order to perform cognitive tasks, children require certain resources, and the types of resources required for each task will differ greatly depending on several factors. For arithmetic calculation, research have studied a number of factors that are proposed to be the main predictors to children's performance. Although it is easy to hypothesize that there is a relation between certain factors and arithmetic articulation, it is not trivial to evaluate the significance of the effects statistically. The goal of this research is to summarize some of the key experimental findings with regards to the connection between working memory and arithmetic calculation.

REVIEW FINDINGS

To provide an understanding of how cognitive factors affect arithmetic calculation, researchers have conducted several experiments on children and on adults of several ages. In review, only a few studies have directly investigated the role of working memory in children's arithmetic calculation performance as pointed out by Berg (2008). The main goal of these experiments is to tie low performance of mathematics or mathematical difficulties as a whole to impairments in few important cognitive measures.

Swanson & Beebe-Frankenberger (2004), conducted a study to assess the contribution of working memory to individual and age-related performance in children's problem-solving performance in first, second and third grade. In their analysis, the authors have identified a structure of tasks that classifies 3 sets of tasks to be traced to short-term memory, working memory and updating. The study distinguishes between the constructs of working memory and short-term memory believed by others (e.g., Diamond, 2013) on the basis that short-term memory measures draw on phonological codes (Salame & Baddeley, 1982), whereas working memory measures draw on resources from the executive system. Furthermore, the study concludes that although working memory is not the exclusive contributor, it plays a major role to perform information integration for problem solving tasks. This is in most part due to the belief that working memory is responsible for holding recent information to make linkage to last inputs received, and also to maintain the meaningful articulation of the problem in hand.

As it is difficult to isolate the performance of one cognitive ability from other abilities, processing speed is often studied to assess children's performance to shed light on its correlation to other factors. In a study performed by Bull & Johnston (1997), children with the mean age of 7 were chosen to participate in a series of designed assessment tests to study relation of short-term memory and processing speed to arithmetic calculation performance. The study highlighted that speed of processing and speed of item identification among two other measures had a significant contribution to a child's performance.

Working memory and phonological processing were studied by Swanson (2004) to explore the contribution of working memory to mathematical problem solving among younger vs. older children of mean 8 and 11 respectively. Chronological age was a significant predictor in the model developed, and it confirmed that a domain-general working memory system does provide notable variance to age-related changes in problem solving. The study has also shown that contribution of working memory to problem-solving accuracy outweighed that of phonological processes. The study also explained ties to long-term memory following Baddeley updated model of working memory, by identifying performance on tasks that relate to calculation, knowledge of algorithms, and knowledge of operations as main measures of that type of memory. This agrees with the argument that working memory will certainly need to retrieve some knowledge in long-term memory in order to articulate and complete the problem-solving tasks (Baddeley, 2001). One of the main findings of this study was on the individual differences between children in the study group, where working memory capacity was connected to availability of cognitive resources. The result stated that children with a larger working memory capacity would have more resources available for storage while representing the problem, in contrast to children with a smaller working memory capacity who might have fewer resources available for the maintenance of information during problem solving. One drawback of this study, in the context of decomposing working memory, is that working memory had to be modeled using a second order model to include component effect in one general factor. This would suppress the possibility to make direct observation on the role of verbal working memory-analogues to phonological loop-or the visual-spatial sketchpad.

Strategy choices in simple and complex addition problems was studied by Geary, et al. (2004) to analyze contributions of working memory and counting knowledge for children with mathematical disability. In their research, the authors studied three main age groups; namely first, third and fifth grade children and two subgroups within each for children known to have mathematical difficulties and another for normal achieving peers. In strategy assessment, the results showed that in simple problems (both operands less than 5) generally children with math difficulties used finger counting more frequently than did their normal peers. With the switch to complex problems (one operand has two digits instead of one), children with math difficulties showed a significant decrease in finger counting and a significant increase in retrieval/guessing strategy with more significance of change in first grader children groups in the two problem types. This of course did not improve accuracy as more errors were present in the complex problems. This result is consistent with previous research by Geary (1991) in showing switch in strategy use in correspondence to complexity of the problem. In agreement with the role of working memory, results for the working memory task indicated that children with math difficulties have a working memory deficit.

Berg (2008) experimented with children in grades 3 to 6; to investigate contributory roles of processing speed, verbal working memory, visual–spatial working memory, short-term memory, and reading. A major advantage of this study in literature is to provide insight on the contribution of individual working memory subcomponents to arithmetic calculation performance, rather than aggregating it as one factor. Results were found to be congruent with previous findings that processing speed and short-term memory do not eliminate the contribution of verbal working memory and visual–spatial working memory to arithmetic calculation. In fact, each of them contributed a unique variance to arithmetic calculation in the presence of all other variables; which supports the notion that they draw on different resources and are independent of other variables. In contrast to other experiments' findings, results did not view processing speed as a controller of arithmetic calculation in the general sample. With these results, strong implications were made regarding role of working memory components, but the author stated that the exact role of each working memory component is still open to question.

In support of the idea that working memory resources are required to solve multistep problems (complex addition of two digit numbers involving carry), Adams and Hitch (1997) researched this relationship in 8 to 11-year-old children using simple and complex mental addition problems. The authors hypothesized that the visual display provides an external record of the addends to supplement working memory, and therefore visual mental addition spans should be higher than oral spans. Results were in agreement with their hypothesis, and another interpretation made is that retrieval is more active in the visually presented problems corresponding to low working memory functioning contrary to orally presented problems which require high working memory functioning.

While these studies provide a great insight on relation of working memory and math, Raghubar, et al., (2010) reviewed evidence from four types of studies to evaluate the proposed relation of working memory and mathematics.: 1) dual task studies establishing the role of working memory during on-line math performance; 2) individual difference studies examining working memory in children with math difficulties; 3) studies of working memory as a predictor of mathematical outcomes; and 4) longitudinal studies of working memory and math. The first two approaches will be discussed in more details and general comments will be made on the latter two.

With regards to dual task experiments, the paper reviews adult experiments and a few studies in children, implying an issue with not having sufficient research on children population. The authors argue that due to dual task nature of the experiment, secondary task will severely affect main task performance. The reasoning behind that argument is that when overlapping of cognitive resources takes place, main task performance will diminish as the secondary task becomes more demanding. Furthermore, in strategy use experiments using dual tasks; it can be noted that children's use of strategy was not impacted by amount of load on working memory. However, the amount of central executive resources used will be affected by the strategy in place. For example, retrieval strategies will have low dependence on such resources compared to verbal or finger counting strategies (Geary et al., 2004).

As for studies relating to children with math difficulties, a general observation made was that some experiments for verbal working memory measures, use non-numerical tests in line with numerical ones to differentiate between children with math difficulties and normal achieving children. Common tests in the literature are the word span backward and digit span backward, where the first one does not involve numerical information in comparison to the latter. The authors mention that digit span backwards has shown a better differentiation between children with math difficulties and those without. Additionally, the effectiveness of backward digit span in distinguishing children with math disabilities versus those without was not always conclusive as contrasted by several studies mentioned in the review. The problem in choosing the effective type of experimental tests or measures becomes more problematic when considering visual-spatial sketchpad subcomponent of the working memory. This is mainly due to differences in age groups being studied in each experiment, the measures used in analyzing role of visual-spatial sketchpad, severity of math difficulty in the subject group and finally the paucity of studies of similar conditions. All of that makes it harder to project predictions of one study on another study with inconsistent parameters. With these findings, the authors state several factors that needs to be understood with regards to relationship between working memory and math: "*age, skill level, language of instruction, the way in which mathematical problems are presented, the type of mathematical skill under consideration and whether that skill is in the process of being acquired, consolidated, or mastered.*" (p. 119).

LeFevre et al. (2005) discuss the relation between working memory and mathematical cognition referenced in The Handbook of Mathematical Cognition (Campbell, 2005), to provide a review for research done regarding subject matter. The authors mention the following on problem complexity: "*There are at least three ways to operationalize problem complexity: (a) operand magnitude (e.g., 1 + 1 vs. 9 + 9); (b) number of digits in the operands (i.e., 2 + 3 vs. 25 + 67); and (c) the presence or absence of carry operations (e.g., 23 + 41 vs. 29 + 46).*" As easy and immaterial it might seem, classification of complexity in this manner is important and aids in interpretation of accuracy and solving times for addition problems. With more steps added to the operation, resource demand increases and the chance of error increases as well. A good reference for studying mental additions is presented in Adams and Hitch (1997).

The concept of *math anxiety* and its interference with the ability to perform math tasks has recently received more attention in the literature. A good reference study was done by Ashcraft & Krause (2007), where the researchers strengthened the idea that anxieties function like a secondary task draining the anxious person's working memory resources.

What are the common measurements used to assess mathematical performance?

Previous research use different measurements in their experiments in an attempt to link disabilities to component-level in working memory. By using these measures, researchers would be able to pinpoint reasons behind making a wrong answer or a slow response, even if accurate, instead of judging only by accuracy of answers. The following is not an exhaustive list; it rather mentions the most common tasks in experiments mentioned in this review.

Visual matrix. The visual matrix task (Swanson, 1995) is a visual-spatial sketchpad measure that assesses each child's ability to recall dots arranged within a matrix.

Mapping. Mapping is also a visual-spatial sketchpad task that requires children to remember a sequence of directions on a map (Swanson, 1992, 1995).

Corsi block task. Originally created by Corsi (1973), the Corsi block task is a visual-spatial sketchpad measure that assesses ability to recall position sequence of blocks on a designated board. Complexity increases gradually by increasing number of blocks in the sequence.

Digit span forward and backward. Digit span tasks are tasks that assess children's ability to remember numerical information embedded in a short sentence either in their stated order or in reverse. This measure is part of several indices developed by Wechsler (1991) and it was first used in the third edition publication in 1991. These indices are measured with respect to a scale known as Wechsler Intelligence Scale for Children (WISC-V; Wechsler, 2014).

Word span forward and backward. Similar to recall testing of digit span, word span is an executive function measure of the working memory, where children are required to mention words instead of numbers presented in a different setting than digit span (La Pointe & Engle 1990).

Listening sentence span. Generally referred to as Listening Span, this test was first used by Daneman and Carpenter (1980) to assess working memory capacity without use of visual representations. It is an executive function measure which demanded the presentation of a set of sentences, uttered loudly, for which children attempted to understand in parallel with the passage and recall the last ending word of each sentence. The number of sentences in the group gradually increased. The working memory capacity was defined as the largest group of ending words recalled.

Strategy assessment. Strategy assessment is usually done by both observing participating children and asking them what were they thinking or how did they come up with their answer. Commonly used strategies are the finger counting, verbal counting without use of fingers, retrieval strategy where no counting takes place and the child gives the answer mostly from his long-term memory, and finally decomposition, which is used in complex addition problem solving.

Mental computation of word problems. Mental computation is a type of task that is presented orally and solved without paper or pencil.

It should be noted that these methods have their pros and cons and they are chosen carefully by the examiner, as not all of these tasks fit the purpose of a certain study. For example, we noticed how a number of these measures is considered to be of non-numerical nature and therefore has its limitations in identifying math disabilities. However, such methods may be superior to numerical measures in extracting performance of verbal working memory. Campbell (2005) worked on a framework for studying mathematical disabilities, which makes a distinction between conceptual and procedural competencies. Previous research can draw on this important framework to categorize problems into conceptual or procedural which aids in identifying which working memory component is responsible for mathematical disability.

DISCUSSION

The investigation of issues related to mathematical cognition and working memory supports the assumption that mathematics and working memory are strongly connected. It has been confirmed by many researchers that working memory is a cognitive resource that is involved in mathematical performance whether it is single- or multi-digit arithmetic. However, complexity issues that arise with problem size and structure need to be taken into account, for the sake of getting better assimilation of developed models and analysis. Processing speed did not appear as a significant contributor in all studies, as pointed out by Berg (2008), mainly because of the difference in age group considered. Chronological age was definitely proven to be one of the significant contributors as children get experienced with strategy selection and develop better switching between retrieval and computational strategies. Gender was accounted for in all mentioned studies (Adams and Hitch, 1997; Geary, et al., 2004; Swanson & Beebe-Frankenberger, 2004; Swanson, 2004; Bull & Johnston, 1997), and was shown to have no statistical significance in young ages of the same age.

As for mathematical difficulties, a consistent finding is that children with mathematical difficulties struggle in retrieving basic arithmetic facts from long-term memory, which adds into slow response and common errors observed with children with mathematical difficulties (Geary, et al., 2004).

The question to be asked is, how can this be utilized in human factors engineering? As it is the case in most of cognition research, understanding the process of cognition, its inputs, outputs and processing gears, aids in a better design of educational systems that accounts for individual differences and detects learning difficulties if present as early as possible. This review did not propose a design for such educational system but it can be used as the first step in understanding the overall directions in literature to identify areas requiring focus and improvement.

There are limitations in the reviewed studies that hinder formulation of one clear understanding of the role of working memory. The first limitation is the difficulty of assessing each component of the working memory by itself, which is usually relaxed by treating working memory system as a unidimensional system as pointed out by Berg (2008). Second, the role of the working memory and even other factors, such as processing speed, has a varying contribution according to chronological age group. The significance of one factor may not maintain same level of significance or contribution when tested on higher age groups. Third, the direct relation of working memory to arithmetic calculation in children was not researched as heavily as other factors, posing a challenge in connecting available literature to one coherent result. These limitations and others constitute a challenge in claiming consistency of results presented but nonetheless, the foundation on which future research is to be built is available and knowledge gaps can be fulfilled with future research work tackling aforementioned limitations. Furthermore, following a framework for studying mathematical difficulties, referenced in Campbell's handbook (2005), would help in achieving more alignment between studies in the literature and opening doors to tracing issues to component level of the working memory model.

Future research could be directed towards involving neuroimaging for working memory specifically for mathematical cognition, having more consistency in task selection in order to have coherent basis for predictive models and taking account of the implications of using numerical versus non-numerical measures in experiments.

As strategy use is considered an important factor in working memory tasks, it is also proposed to provide more studies on strategy selection and discovery and how the brain processes mathematical information at different ages and ability levels (Raghubar, et al., 2010).

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